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A-deformations of a surface with stationary lengths of LGT-lines. (English summary)
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The authors study infinitesimal deformations of a surface that preserve the area element of the surface, which are called A-deformations. They define a line of geodesic torsion (LGT-line) to be a line on a surface whose direction coincides at every point with a principal direction of geodesic torsion. Then these two notions are combined and A-deformations of a surface that preserve the length of lines of geodesic torsion on the surface are examined.

First, a necessary and sufficient condition for the preservation of lengths of LGT-lines under an A-deformation of a surface is proved. Namely, this condition is the equality

$$2T^{\gamma\delta}c_{\alpha\gamma}g_{\beta\delta}+2T^{\gamma\delta}c_{\beta\gamma}g_{\alpha\delta}=2\mu(b_{\alpha\beta}-Hg_{\alpha\beta}),$$

where $\mu(x^1,x^2)$ is a certain function. For a surface with parameter equation $\mathbf{r}=\mathbf{r}(x^1,x^2)$, the $T^{\gamma\delta}$ are deformation tensors on the surface. These are determined by the partial derivatives of the displacement vector $\mathbf{y}(x^1,x^2)$, which in the basis $\mathbf{r}_1$, $\mathbf{r}_2$, $\mathbf{n}$ are as follows:

$$\mathbf{y}_i=c_{i\alpha}T^{\alpha\beta}\mathbf{r}_\beta+c_{i\alpha}T^\alpha\mathbf{n}.$$

The $b_{\alpha\beta}$ are the coefficients of the second fundamental form, $c_{i\alpha}$ is the discriminant tensor of the surface ($c_{11}=c_{22}=0$, $c_{12}=-c_{21}=\sqrt{g}$ and $g=g_{11}g_{22}-g_{12}^2$) and H is the mean curvature of the surface. The $g_{\alpha\beta}$ are the coefficients of the first fundamental form and all the indices take values 1 and 2.

Then the special cases of a trivial A-deformation, an A-deformation of a minimal surface and an A-deformation of a non-flat, non-minimal surface that preserves the lengths of LGT-lines are examined. In each of the last two cases, the existence is shown of a nontrivial A-deformation with the required property. In the case of a minimal surface, the connection with an A-deformation of the surface that preserves the length of asymptotic lines is mentioned.

Also, A-deformations with stationary length of LGT-lines for surfaces of elliptic and of hyperbolic type are considered. Again, existence results are stated.

In the last section, the deformation tensors of an A-deformation that preserves the length of LGT-lines are calculated explicitly for an elliptic paraboloid.

Where suitable, the authors point out the similarities and differences between infinitesimal bendings of a surface and A-deformations since the latter generalize the former.

Reviewed by *Wendy Goemans*

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